A Continuous Model of Hybrid Zone Dominated by Assortative Mating

— IRT 1 Project —

常龙啸 (Chang Longxiao)
Supervisor: Prof. Dr. Dirk Metzler

EES program, LMU
Longxiao.Chang@campus.lmu.de

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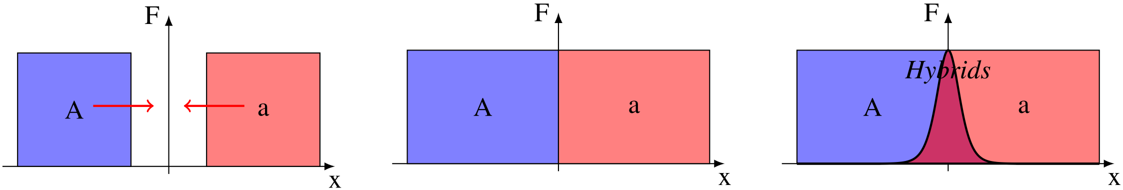
Contents

A Brief Introduction to Hybrid Zone

Hybrid Zone

A hybrid zone is an area where genetically distinct populations come into contact.

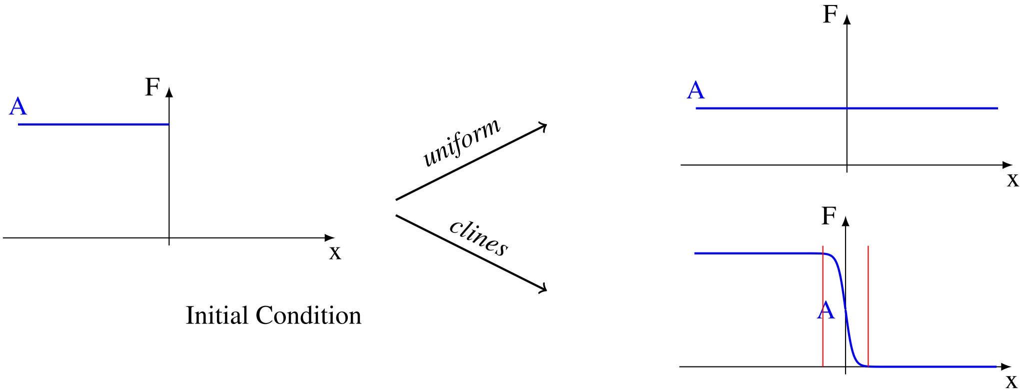


Isolation   Secodary Contact   Hybrid Zone

1

[1] N H Barton and G M Hewitt (Nov. 1985). "Analysis of Hybrid Zones". en. In: Annual Review of Ecology and Systematics 16.1, pp. 113–148. ISSN: 0066-4162. DOI: 10.1146/annurev.es.16.110185.000553. URL: https://www.annualreviews.org/doi/10.1146/annurev.es.16.110185.000553 (visited on 07/29/2024).

Clines



If the selection on hybrids can balance the homogenization caused by dispersal, sigmoid clines will be formed in the hybrid zone, showing a rapid change of allele frequency in a narrow area.[2]

[2] N. H. Barton and G. M. Hewitt (Oct. 1989). “Adaptation, speciation and hybrid zones”. en. In: Nature 341.6242, pp. 497–503. ISSN: 0028-0836, 1476-4687. DOI: 10.1038/341497a0. URL: https://www.nature.com/articles/341497a0 (visited on 05/07/2024).

PDE as continuous model

Continuous Model

Description of a continuous system (u) with x and t:

$$u_t = \frac{\partial u}{\partial t}, u_{tt} = \frac{\partial^2 u}{\partial t^2} \dots$$
$$u_x = \frac{\partial u}{\partial x}, u_{xx} = \frac{\partial^2 u}{\partial t^2} \dots$$

PDE (partial differential equation)

$$f(u_t, u_{tt}, u_x, u_{xx}, x, t) = 0$$

As for hybrid zone modelling:

$$\frac{\partial p}{\partial t} = dispersal + selection$$

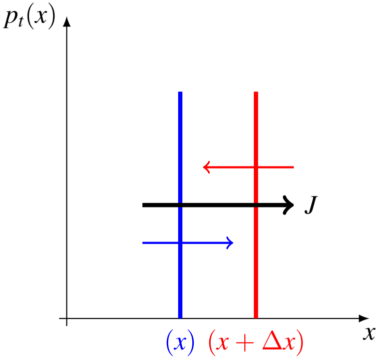
Dispersal: Diffusion Function

How does the population distribution change over time by dispersal?

$$\frac{\partial p(x,t)}{\partial t} = ?$$

Considering the population diffuse along the area, in which D is the diffusion rate.
Diffusion model:

$$\frac{\partial p}{\partial t} = D\frac{\partial^2 p}{\partial x^2}$$



Diffusion Model and Brownian Motion

Diffusion model:

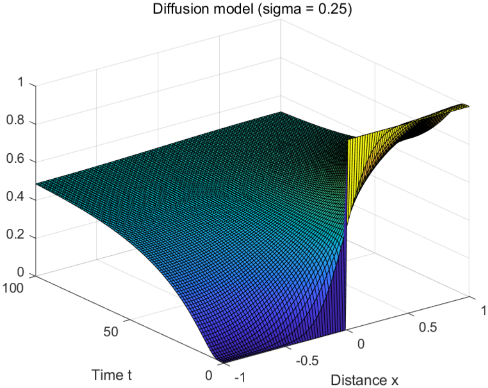
$$\frac{\partial p}{\partial t} = D\frac{\partial^2 p}{\partial x^2}$$

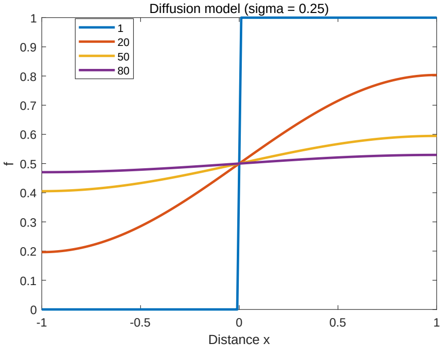
The analytical solution is:

$$p(x,t) = \frac{p_0}{\sqrt{4\pi Dt}} e^{-\frac{x^2}{4Dt}}$$

which follows the normal distribution $N(0, 2Dt)$.
Also, the density distribution of the Brownian motionat t also follows the same distribution, which provide another explanation.

Test Diffusion Model





Tension Zone Model

Natural selection on hybrids:

Table: Fitness of different genotypes (1 locus)

	AA	Aa	aa
frequency(p)	p^2	$2pq$	q^2
fitness	1	$1-s$	1
frequency(f1)	p^2	$2pq(1-s)$	q^2

(p is the frequency of allele A, q is the frequency of a, s is the selection on hybrids) [3][4]

the tension zone model:

$$\frac{\partial p}{\partial t} = \frac{\sigma^2}{4}\frac{\partial^2 p}{\partial x^2} - spq(p-q)$$

(parabolic partial differential equations)
When it becomes to a constant condition, $\frac{\partial p}{\partial t} = 0$,then we have:

$$\frac{d^2 p}{dx^2} = \frac{4}{\sigma^2} sp(1-p)(1-2p)......(p+q=1)$$

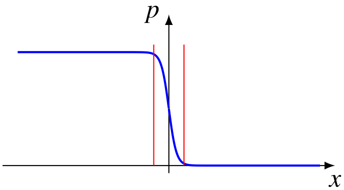
[3]N H Barton (Dec. 1979). "The dynamics of hybrid zones". en. In: Heredity 43.3, pp. 341–359. ISSN: 0018-067X, 1365-2540. DOI: 10.1038/hdy.1979.87. URL: https://www.nature.com/articles/hdy197987 (visited on 05/07/2024).

[4]A. D. Bazykin (Dec. 1969). "HYPOTHETICAL MECHANISM OF SPECIATION". en. In: Evolution 23.4, pp. 685–687. ISSN: 00143820. DOI: 10.1111/j.1558-5646.1969.tb03550.x. URL: https://academic.oup.com/evolut/article/23/4/685/6867639 (visited on 05/07/2024).

Tension Zone Model

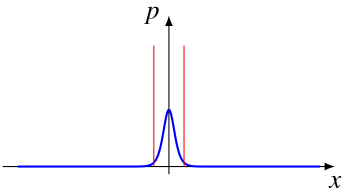
p_1

$$p_1 = \frac{1}{1 + e^{\frac{2\sqrt{s}}{\sigma} ax}}$$

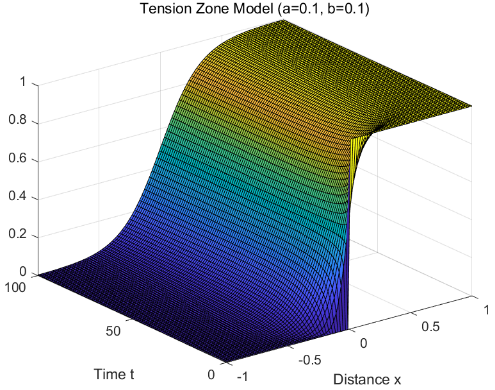


p_H

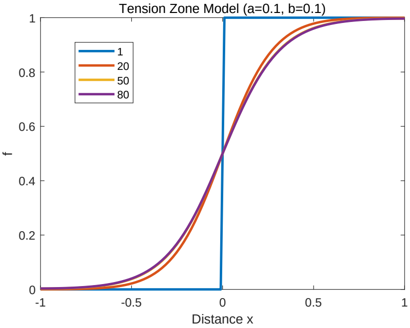
$$p_H = \frac{2(1-s)e^{\frac{2\sqrt{s}}{\sigma} a(x+\xi)}}{\left(1 + e^{\frac{2\sqrt{s}}{\sigma} a(x+\xi)}\right)^2}$$



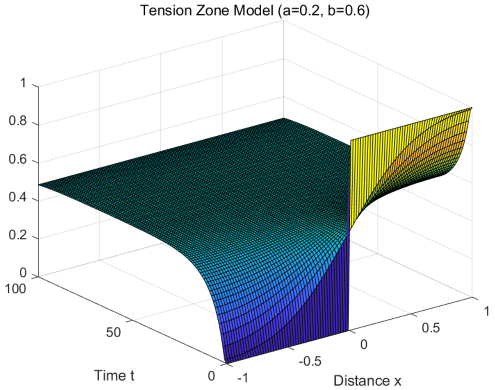
Simulation of Tension Zone



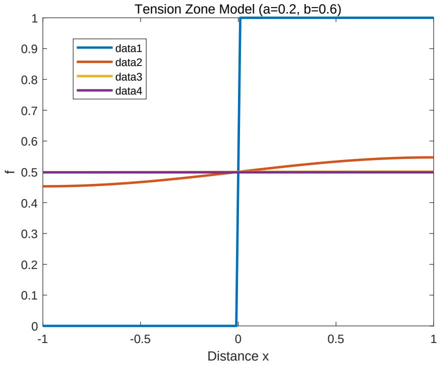
$$b = \sigma; a = s$$



Simulation of Tension Zone



$$b = \sigma ; a = s$$



Assortative Mating: Sexual Selection

The selection happens before reproduction (without HWE)
Lack of the constraint between p_H and p, $p_H = 2 \cdot p(1-p)$

A simple model of 2 alleles (A and a) in 1 locus

	AA	Aa	aa
f	p-m	2m	q-m

The probability that two genotypes mating with each other is limited by a mating matrix:

Table: Mating Matrix with 1 locus

	AA	Aa	aa
AA	1	1-a	1-f(a,b)
Aa		1	1-b
aa			1

Assortative Mating in Tension Zone

To simplify, we considering a symmetrical mating matrix:

Table: simplified mating matrix with 1 locus

	AA	Aa	aa
AA	1	1-r	1-R
Aa		1	1-r
aa			1

Set the frequency of dominant homozygotes as p_D, recessive homozygotes as p_R, heterozygotes as p_H

$$p'_D = \frac{p^2 - 2rpm + 2rm^2}{z}$$

$$p'_R = \frac{q^2 - 2rqm + 2rm^2}{z}$$

$$p'_H = \frac{2(R-r)m + 2(4-2r-R)m^2 + 2(1-R)pq}{z}$$

$$z = 1 - 2(2r-R)m - 2(R-4r)m^2$$

in which, z is used to normalize the frequencies (keeps $p'_D + p'_R + p'_H = 1$).

Analysis of Assortative Mating

$$p'_H = \frac{(1-r)(2m-4m^2)+2m^2+2(1-R)(pq-m+m^2)}{1-2r(2m-4m^2)-2R(pq-m+m^2)}$$

with the following two approximation

$$R \simeq 4r$$

$$pq \simeq m$$

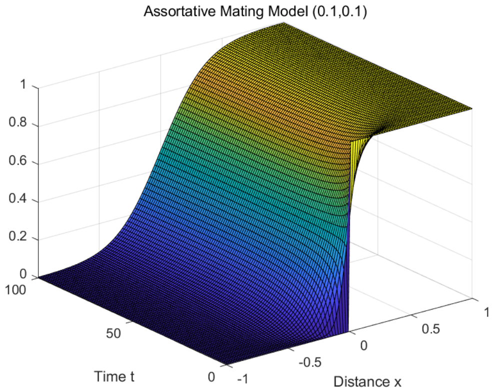
As $p_H = 2m$, we could simplify the formula to:

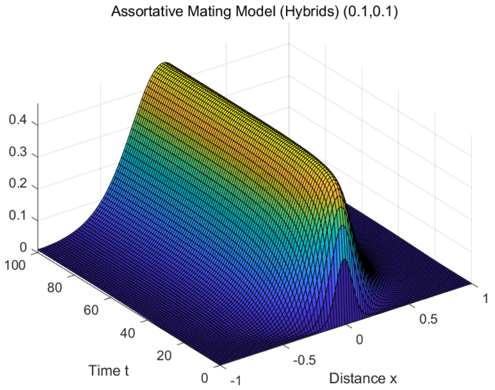
$$p'_H = \frac{p_H(1-r-rp_H)}{1-2rp_H}$$

the dynamic of hybrids based on sexual selection could be represented by:

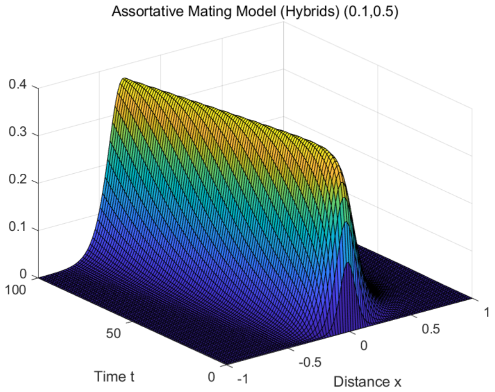
$$\frac{\partial p_H}{\partial t} = \frac{\sigma^2}{4}\frac{\partial^2 p_H}{\partial x^2} + \frac{rp_H(p_H-1)}{1-2rp_H}$$

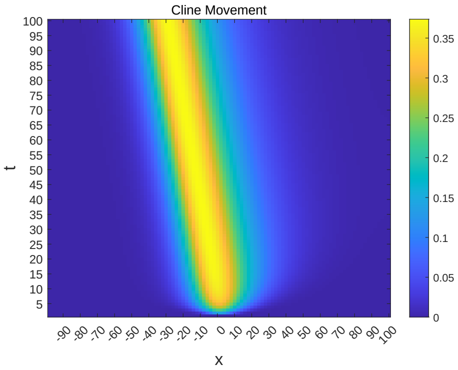
Assortative Mating and Clines



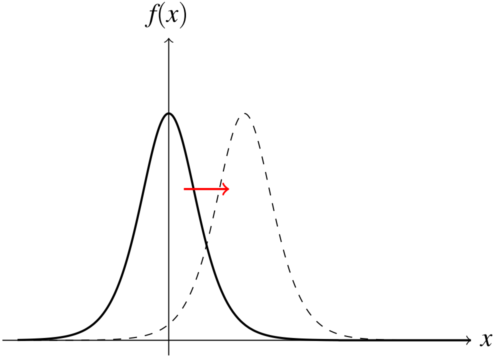


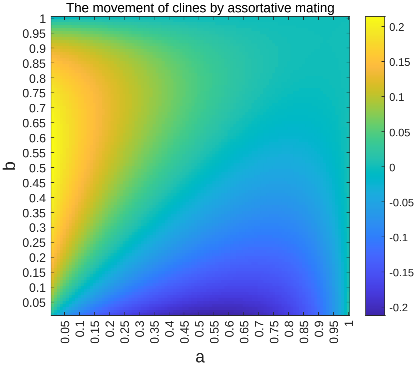
Cline Movement



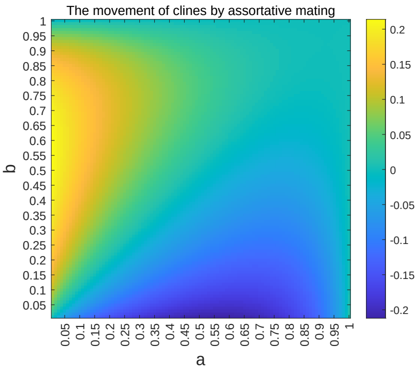


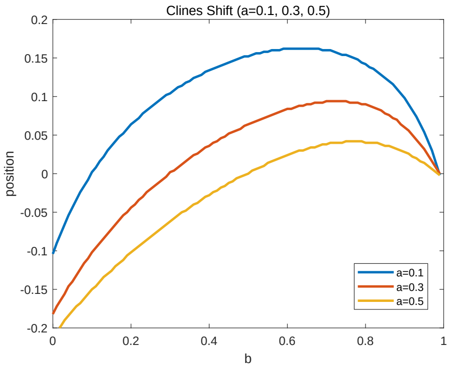
Cline Movement



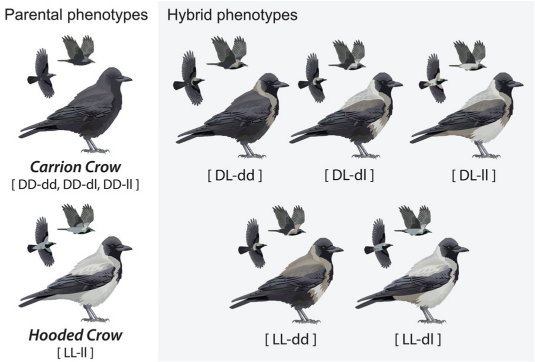


Cline Movement

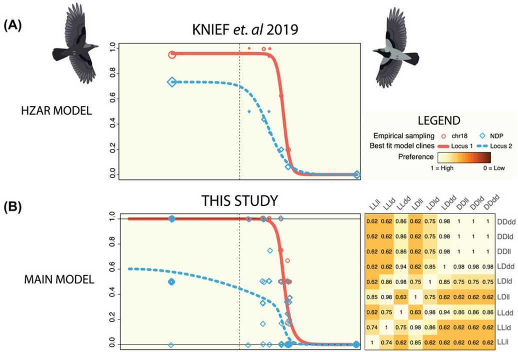




Crow Population in Hybrid Zone



[5]



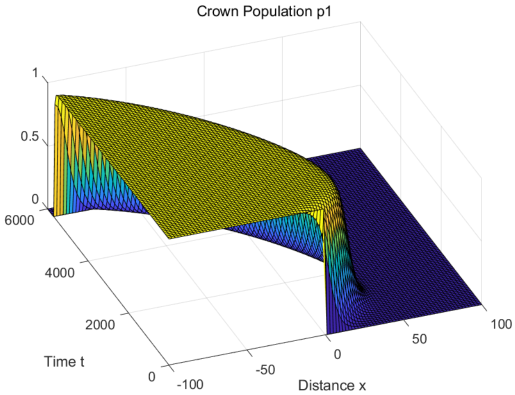
[5] Dirk Metzler et al. (Dec. 2021). "Assortative mating and epistatic mating-trait architecture induce complex movement of the crow hybrid zone". In: Evolution 75.12. Publisher: John Wiley & Sons, Ltd, pp. 3154–3174. ISSN: 0014-3820. DOI: 10.1111/evo.14386. URL: https://onlinelibrary.wiley.com/doi/full/10.1111/evo.14386.

Simulating Crow Population

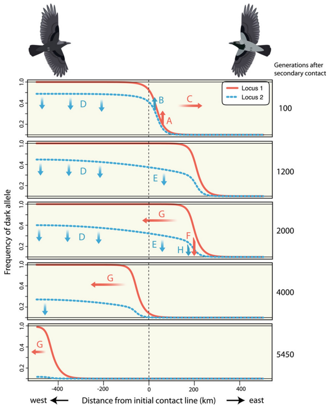
$$\frac{\partial \mathbf{p}}{\partial t} = \frac{\sigma^2}{4} \frac{\partial^2 \mathbf{p}}{\partial x^2} + \mathbf{p}' - \mathbf{p}$$

Using the data by Siefke 1994 and Metzler et al. 2021. Distance unit is 5 km (the width of block), time unit is 6 years (generation time).

$\sigma = 14.9$ by normal distribution fitting. To simplify, we use $\sigma = 3(\textit{distance unit})$.



Simulating Crow Population



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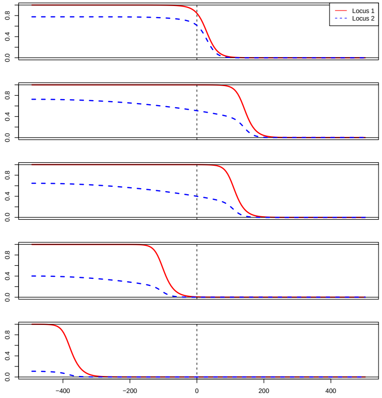

[a] Dirk Metzler et al. (Dec. 2021). "Assortative mating and epistatic mating-trait architecture induce complex movement of the crow hybrid zone". In: Evolution 75.12. Publisher: John Wiley & Sons, Ltd, pp. 3154–3174. ISSN: 0014-3820. DOI: 10.1111/evo.14386. URL:

Discrete Model and Continuous Model

$$x(t) = f(x(t-1), x(t-2), \dots)$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = f(x, t)$$

$$f_{x,g}(t+1) = c_x(t) \cdot \sum_{y,z,g_1,g_2} \mu(g|g_1, g_2) \cdot f_{y,g_1}(t) \cdot d_{y,x} \cdot f_{z,g_1}(t) \cdot d_{z,x} \cdot w_{g_1,g_2}$$

$$\frac{\partial \mathbf{p}}{\partial t} = D \frac{\partial^2 \mathbf{p}}{\partial x^2} + \mathbf{p}' - \mathbf{p}$$

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