Will a sequencial assembled community be more stable than a random one?

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A Brief Review

Stability-Biodiversity as the Main Question in Ecology

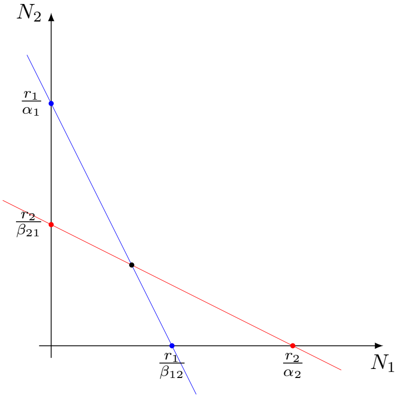
The mechanisms underlying **species coexistence** and **biodiversity maintenance** have been central questions in ecology. Community stability, as a fundamental aspect of biodiversity maintenance, has been extensively studied, yet no unified framework currently exists to fully describe its relationship with diversity across different ecological systems.

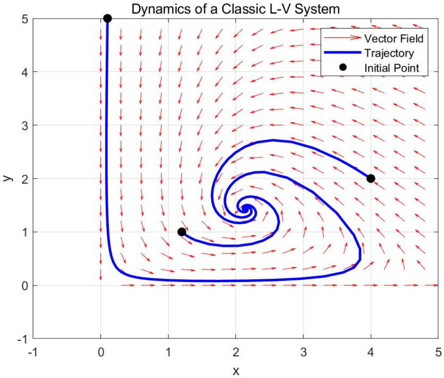
The traditional view often holds that biodiversity is the foundation of ecosystem stability. A large number of experiments and surveys have confirmed that the loss of biodiversity disrupts the maintenance of ecosystems.

Related theoretical studies typically approach the problem from the perspective of dynamical systems.
Biological communities are often modeled using a generalized Lotka-Volterra framework:

$$\frac{\mathrm{d}N_i}{\mathrm{d}t} = N_i \left(r_i + \sum_{j=1}^{S} \alpha_{ij} N_j \right) \tag{1}$$

.
The second term in the equation may have different variants (to model various ecological effects) or include time-varying coefficients. The matrix formed by α_{ij} — the interaction matrix $\boldsymbol{A}$ — is also represented as a network. As a natural extension of food web models, it is studied using methods from graph theory.


N_2
$\frac{r_1}{\alpha_1}$
$\frac{r_2}{\beta_{21}}$
$\frac{r_1}{\beta_{12}}$
$\frac{r_2}{\alpha_2}$
N_1

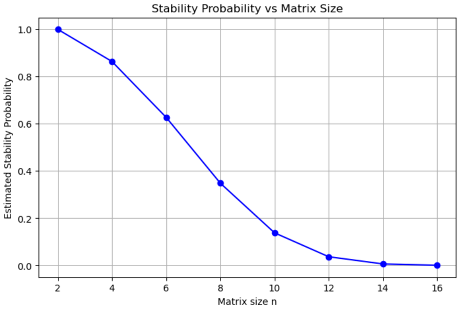

Dynamics of a Classic L-V System
Vector Field
Trajectory
Initial Point
y
x
-1
0
1
2
3
4
5

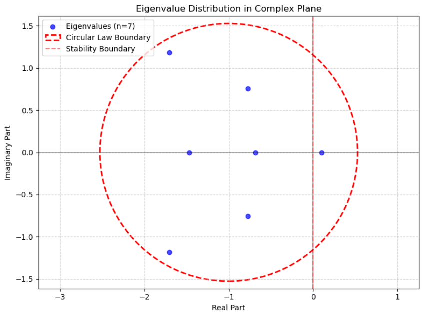
May (1972) [1] investigated the linear approximation of the system (equation 1) near its stable equilibrium:

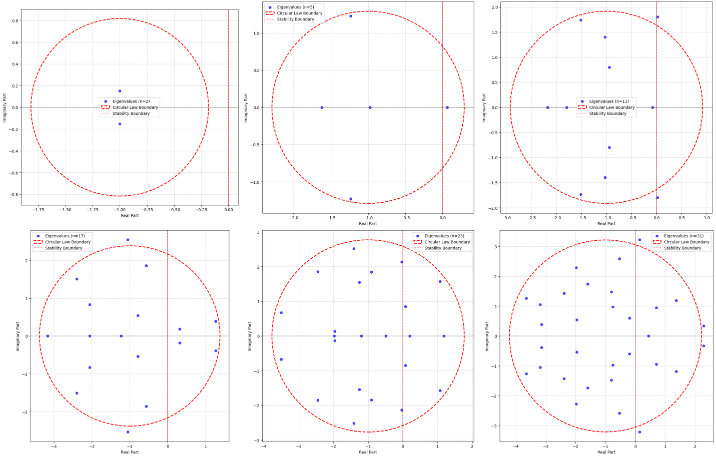
$$\frac{\mathrm{d}\boldsymbol{A}}{\mathrm{d}t} \approx \boldsymbol{J}\boldsymbol{N} \tag{2}$$

where $\boldsymbol{A}$ is a random matrix.
The result shows that such a system is stable only if $\alpha < (n)^{-1/2}$, where α is the average interaction strength, n is the number of species.

[1] "Will a Large Complex System be Stable?",May (1972)


Stability Probability vs Matrix Size
Estimated Stability Probability
Matrix size n
1.0
0.8
0.6
0.4
0.2
0.0
2
4
6
8
10
12
14
16

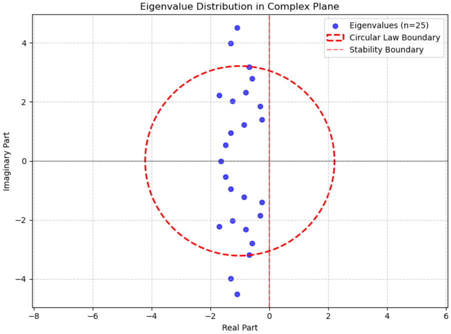

Eigenvalue Distribution in Complex Plane
Eigenvalues (n=7)
Circular Law Boundary
Stability Boundary
Imaginary Part
Real Part
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−3
−2
−1
0
1


Eigenvalues (n=2)
Circular Law Boundary
Stability Boundary
Eigenvalues (n=5)
Eigenvalues (n=11)
Eigenvalues (n=17)
Eigenvalues (n=23)
Eigenvalues (n=31)
Real Part
Imaginary Part

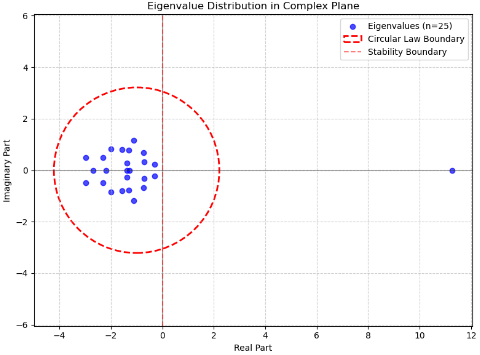
One explanation for the contradiction in the diversity-stability relationship is that real community structures are constrained, and the interaction matrix is not a completely random uniform distribution.

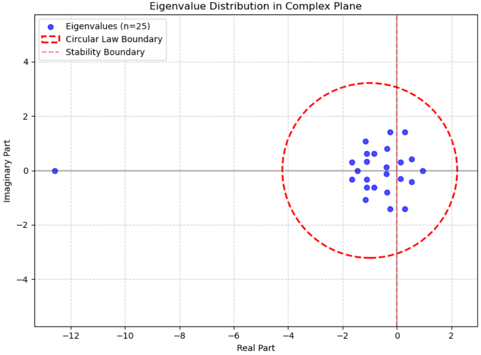
For example, the **types of interactions**, the **connectivity patterns** of the interaction network, and the **feasibility conditions** of the community all impose constraints on community structure.

Allesina and Tang (2012) [2] extended May's results to well-defined interactions (for example predator-prey, mutualistic or competitive) and found remarkable differences between predator-prey interactions, which are stabilizing, and mutualistic and competitive interactions, which are destabilizing.



[2] "Stability criteria for complex ecosystems", Allesina and Tang (2012)


Eigenvalue Distribution in Complex Plane
Eigenvalues (n=25)
Circular Law Boundary
Stability Boundary
Imaginary Part
Real Part


Eigenvalue Distribution in Complex Plane
Eigenvalues (n=25)
Circular Law Boundary
Stability Boundary
Imaginary Part
Real Part

Rohr et al. (2014) [3] noted that for a system with a given structure, feasibility conditions

$$D(\beta) = \{\mathbf{r} = N_1^* \mathbf{v}_1 + N_2^* \mathbf{v}_2 + \cdots + N_S^* \mathbf{v}_S \mid N_1^* > 0, N_2^* > 0, \ldots, N_S^* > 0\}, \quad (3)$$

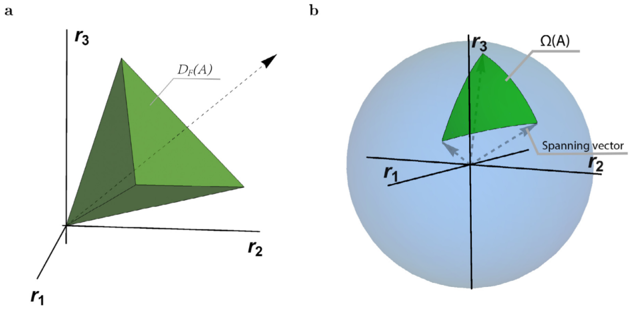
are not always satisfied.

By combining feasibility and stability conditions, they demonstrated that observed network structures tend to exhibit higher structural stability $\omega(\beta) = \frac{V(D(\beta) \cap B_S)}{V(B_S)}$.

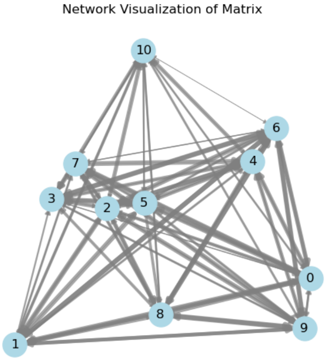
Song and Saavedra (2018) [4] discussed the diversity-stability relationship based on the structural stability framework again.

[3] "On the structural stability of mutualistic systems", Rohr et al. (2014)

[4] "Will a small randomly assembled community be feasible and stable?", Song and Saavedra (2018)


a
r_3
$D_F(A)$
r_2
r_1
b
r_3
Ω(A)
Spanning vector
r_1
r_2

Moreover, from the perspective of interaction networks, specific network structures may serve as sources of stability, for example, Bastolla et al. (2009) [5]



[5] "The architecture of mutualistic networks minimizes competition and increases biodiversity", Bastolla et al. (2009)

Coexistence as the Result of Assembly

Theoretical analyses indicate that the likelihood of a randomly assembled set of species stably coexisting is low. Various modeling constraints on the distribution of interactions can partly explain species coexistence and community stability, but they rarely provide mechanistic explanations.
However, from another perspective, Serván et al. (2018) [6] proves that given a random set of species, the system is very likely to eventually evolve toward stable coexistence of a subset of species.

[6] "Coexistence of many species in random ecosystems", Serván et al. (2018)

We can note that the dynamic evolution of communities provides a mechanism guiding stability.

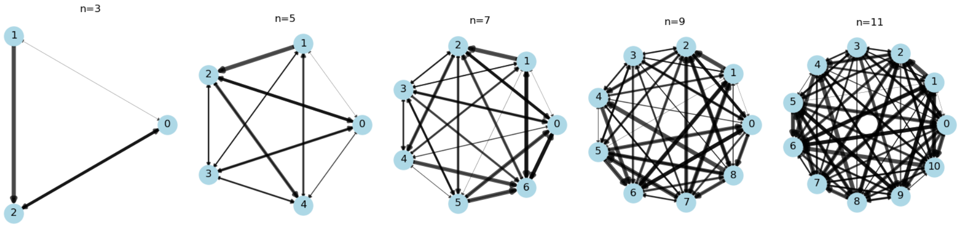
Just as evolutionary theory points out that genes with high fitness are retained under natural selection — explaining the observation of high-fitness traits — we can similarly say that only stable communities persist, which explains why observed communities exhibit high stability.

Therefore, the key question is: what kind of **community assembly mechanisms** (that lead to the observed community structures) give rise to stability? And what are the characteristics of such assembly processes?

A mathematically intriguing approach is the random walk on a hypergraph.
Angulo et al. (2021) [7] introduced a formalism based on algebraic topology and homology theory to study the space of species coexistence formed by a given pool of species.
The assembly and disassembly of ecological systems are then a (random) path on the hypergraph.

[7] "Coexistence holes characterize the assembly and disassembly of multispecies systems", Angulo et al. (2021)

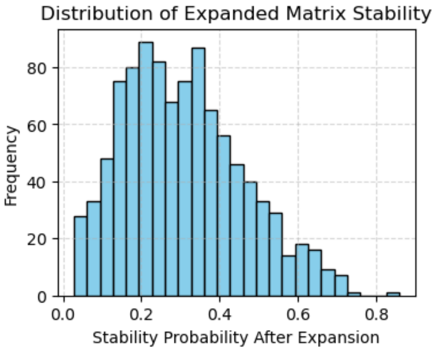
A more straightforward idea is to still base the analysis on random matrices while incorporating system evolution. Just as simple models of network evolution—such as the scale-free network model—generate degree distributions that random networks with a fixed number of nodes do not exhibit, we consider simulating community evolutionary dynamics through a method of randomly expanding stable matrices.

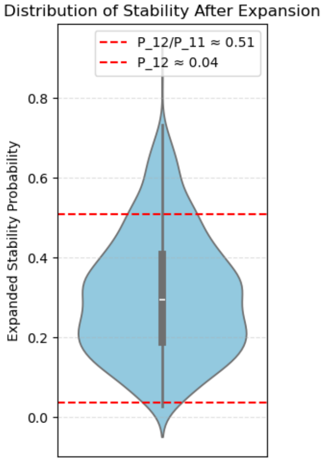


We considering the following (real-value) network construction model based on network stability. With a certain dynamics pattern, the system start from a single node or a small group of nodes. Randomly add a node with random interaction strength, the new node is kept only if the new system with it is also stable.
Basically, we are asking the following question: if here is an $\boldsymbol{A}$ so that system (equation 2) is stable, what is the probability that an extension $\boldsymbol{A}^*$ also lead to a stable system $\Pr(\boldsymbol{A}^+\text{stable}|\boldsymbol{A}\text{stable})$, where

$$\boldsymbol{A}^+ = \begin{bmatrix} \boldsymbol{A} & b \\ c^T & d \end{bmatrix} \tag{4}$$

, b, c, d are random vector/elements?

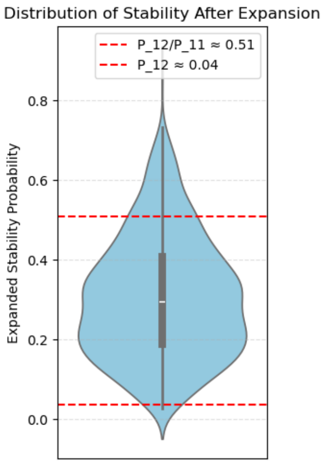

Distribution of Expanded Matrix Stability
Frequency
0
20
40
60
80
0.0
0.2
0.4
0.6
0.8
Stability Probability After Expansion

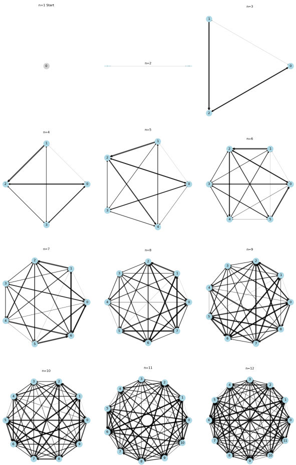

Distribution of Stability After Expansion
P_12/P_11 ≈ 0.51
P_12 ≈ 0.04
Expanded Stability Probability
0.0
0.2
0.4
0.6
0.8

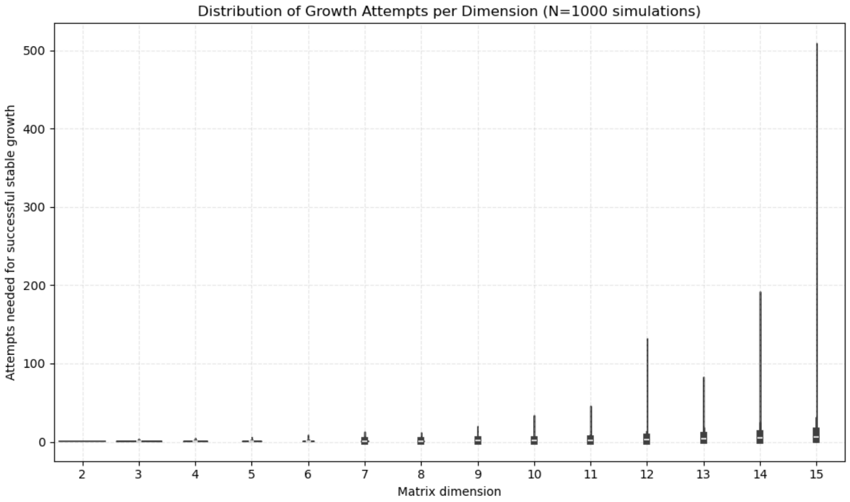
$$\frac{\Pr(\boldsymbol{A}_{12}\ \text{stable})}{\Pr(\boldsymbol{A}_{11}\ \text{stable})} >$$

$$\Pr(\boldsymbol{A}_{11}^{+}\ \text{stable}|\boldsymbol{A}_{11}\ \text{stable}) >$$

$$\Pr(\boldsymbol{A}_{12}\ \text{stable})$$




n=1 Start
n=2
n=3
n=4
n=5
n=6
n=7
n=8
n=9
n=10
n=11
n=12


Distribution of Growth Attempts per Dimension (N=1000 simulations)
Attempts needed for successful stable growth
0
100
200
300
400
500
2
3
4
5
6
7
8
9
10
11
12
13
14
15
Matrix dimension

Some questions

- How to organize the simulations
- Analytical result of the process
- Environment constraints
- Back to the dynamic model: intrinsic rate and population size

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